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CORE

DISCUSSION PAPER

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**Aging, social security design
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Abstract

This paper analyzes the impact of aging on capital accumulation and welfare in a country with a sizable unfunded social security system. Using a two-period overlapping-generation model with endogenous retirement decisions, we show that both the type of aging and the type of unfunded social security system are important in understanding this impact. We consider two demographic changes, declining fertility and increasing longevity, and three types of pensions, defined contributions, defined benefits and defined annuities, to investigate the differences in implications of aging.

Keywords: aging, public finance sustainability, social security

JEL classification: H2, F42, H8.

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1 Introduction

Demographic aging poses a major challenge to all industrialized economies and to a number of developing countries. Driven by two concomitant factors, an ever increasing longevity and a sharp decline in fertility, aging implies a raise in the ratio of elderly to the rest of population, the so-called dependency rate. This transformation is expected to have a number of consequences, which are not always understood in public discussions. Some consequences are clearly unfavorable such as the effect of increasing dependency on financial balance of unfunded pensions. Some others are perceived to be positive: if the decline in fertility outweighs the increase in longevity we would have a decrease in total population, which may be welcomed by some on the basis of environmental concerns. Finally, there are some ambiguous effects. An example is the effect of aging on capital accumulation, which is one the key factors of growth. Studying the effect of aging on capital accumulation is particularly difficult when a number of economic dimensions with substantial age-dependent public presence are included. The most common cases of such dimensions include national debt, long term care, and unfunded social security.

In this paper, we use a two period overlapping generation model, which enables us to assess the levels of capital accumulation and welfare in a society subject to two types of demographic change: a decline in fertility and an increase in longevity. These two changes lead to aging in the society, defined as an increase in the ratio of old population to young population. In the absence of a change in policies, aging can put a substantial stress on the sustainability of public finances.

In comparison to the canonical Diamond's model, our model will comprise a number of additional features that make it more realistic, and can lend itself to calibration.

- Endogenous age of retirement.
- Pay-as-you go social security that can be either defined benefits, defined contributions or defined annuities.

The main contribution of this study is to analyze the incidence of these two factors on the effects of aging on capital accumulation and social welfare. We show that this incidence depends on the type of social security (funded or not, defined benefits or defined contributions), and on the way the retirement decision is regulated (optimal or early retirement).

The aging of the population presents a major challenge for most OECD member countries. Whether or not the combination of increased longevity and a reduced

birth rate will directly reduce the growth rates of these economies by slowing the growth of the capital stock and by weakening the productivity of the labor force is an open question. There exist a large number of studies devoted to this question. Some deal with aging without making the distinction between longevity and fertility changes; other make that distinction. Some are empirical; others are theoretical quite often accompanied by calibrated simulations. Some are assuming away pensions and other focus on them. Another distinction is between models of exogenous versus endogenous growth; in the latter, the way human capital formation adjusts to aging labor force plays a key role. Relative to this impressive amount of work, our paper compares different types of social security systems and distinguishes between longevity increase and fertility decline. In that respect it is at odds with the existing literature. We cannot survey this literature; we just provide a short overview.

Among the studies on increased longevity, there is Bloom et al. (2003) who indicate that this demographic change results in more capital accumulation even if retirement is endogenous. Echevarria (2002) reaches the same conclusion. Kalemli-Ozcam et al. (2000) show that the positive effect of mortality decline is made larger if education adjusts. De la Croix and Licandro (1999) and Zhang et al. (2001, 2003) argue that the effect of increasing longevity depends on its initial level. For low levels of life expectancy the effect is positive but it can turn negative for high levels. Note also the empirical finding of Kinugasa and Mason (2006) who explain the increase of wealth across countries by the mortality decline.

Turning to studies devoted to fertility, there are the papers by Prettnner and Prskawetz (2010a,b) who point out that a country with a lower fertility rate is able to sustain higher levels of per capita income than a country with high fertility. The authors use the framework of an endogenous growth model.

Finally, several papers study the effect of aging on income and growth in a setting of open economies that age at contrasting paces. Börsch-Supan et al. (2005) show that that capital flows from fast-aging regions to the rest of the world is initially substantial but that trends are reversed when households decumulate savings. They also conclude that closed-economy models of pension reforms miss quantitatively important effects of capital mobility. Vogel et al. (2012) analyze the reform of pensions in an integrated economy. Their main point is that the best way to cope with aging is through an increase in the retirement age and in human capital investment.

The rest of the paper is organized as follows. In section 2, we present the basic model and the main results for an economy that consists of identical individuals with a defined contribution pension system. Section 3 is devoted to a comparison

of defined benefits and defined contributions pensions systems. A fourth section is devoted to the dynamics of this model. As it appears clearly the demographic shock will differ depending on the type of social security and on the driver of aging.

2 The basic model

We use a standard two-period overlapping generation model. An individual belonging to generation t lives in two periods t and $t + 1$. The first period of her life has a unitary length, while the second one has a length $\ell \leq 1$, where ℓ reflects variable longevity. In the first period, the individual works and earns w_t which is devoted to the first-period consumption, c_t , saving s_t and pension contribution τ . In the second period, she works an amount of time $z_{t+1} \leq \ell \leq 1$ and earns $z_{t+1}w_{t+1}$. This earning, the proceeds of saving $R_{t+1}s_t$ and the PAYG pension p finances the second period consumption d_{t+1} . We assume that working in the second period z_{t+1} implies a monetary disutility $v(z_{t+1}, \ell)$, with $\frac{\partial v}{\partial z} > 0$, $\frac{\partial^2 v}{\partial \ell^2} > 0$ for the existence of a unique solution, and $\frac{\partial v}{\partial \ell} < 0$, which reflects the idea that an increase in longevity fosters later retirement. Note that, for simplicity, the earnings in the second period of life is not taxed. Intuitively, the end of the first period can be interpreted as the statutory age of retirement. We also abstract from modeling the funded social security system by assuming it is identical to the standard savings. Thus, the pension contribution parameter τ measures the relative size of the unfunded pensions. In other words, $\tau = 0$ implies that the whole pension system is funded.

Denoting by $u(\cdot)$ the utility function for consumption c or d and U the lifetime utility, the problem of an individual of generation t is:

$$\max U_t = u(w_t - \tau - s_t) + \beta \ell u\left(\frac{w_{t+1}z_{t+1} + R_{t+1}s_t + p - v(z_{t+1}, \ell)}{\ell}\right) \quad (1)$$

where $p = \tau(1 + n)$ is the pension benefit in period $t + 1$ and β is the time discount factor. The gross rate of population growth $(1 + n)$ is equivalent to the number of children per individual in this set up.

The first order conditions for life time utility maximization are simply given by:

$$v'_{z_{t+1}}(z_{t+1}, \ell) = w_{t+1} \quad (2)$$

$$\beta R_{t+1} u'(\tilde{d}_{t+1}) - u'(c_t) = 0 \quad (3)$$

where $\tilde{d}_{t+1} = d_{t+1} - v(z_{t+1}, \ell)$. The first condition (2) shows that the marginal monetary disutility from second period work needs to be equal to the wage rate at the optimum. The second condition is the consumption Euler equation, and it shows that the individual cannot gain further utility by reallocating consumption between periods. In order to be able to show some of our results analytically, we will use simple functional forms for $u(\cdot)$ and $v(\cdot)$. Accordingly, we assume that the period utility function is logarithmic $u(x) = \ln x$, and the monetary disutility function is quadratic in the main argument $v(x) = x^2/2\gamma\ell$. One clearly sees from the latter functional form that the disutility of working longer can be mitigated by an increase in longevity. We can now re-write the problem of the individual as the following maximization of:

$$U_t = \ln(w_t - \tau - s_t) + \beta\ell \ln\left(\frac{R_{t+1}s_t + z^2/2\gamma\ell + p_t}{\ell}\right) \quad (4)$$

where $p_t = \tau(1+n)$ ¹. The first order condition with respect to z_{t+1} yields :

$$z_{t+1} = z_{t+1}^* = \gamma\ell w_{t+1} \quad (5)$$

where, $*$ denotes an optimal solution. Using this optimality condition, and incorporating $p_t = \tau(1+n)$, we get an explicit solution for optimal saving rates from the first order condition with respect to s_t :

$$s_t = \frac{\beta\ell}{1+\beta\ell}w_t - \frac{\gamma\ell w_{t+1}^2}{2R_{t+1}(1+\beta\ell)} - \tau\left(\frac{\beta\ell}{1+\beta\ell} + \frac{1+n}{(1+\beta\ell)R_{t+1}}\right) \quad (6)$$

In many countries, z is not the outcome of a choice without a distortion. Through an array of programs, workers are induced to retire at ages different from what they would choose in the absence of these programs. We consider a case where the workers are induced to retire earlier than they wish to do so, and denote this induced early retirement by \bar{z}^2 . In the case of early retirement, we rewrite equations

¹As seen below, this implies defined contributions.

²An alternative specification could be that second period labor is subject to a proportional tax θ whose proceeds are returned to the old workers. Their problem would be to choose z such as to maximize :

$$wz(1-\theta) + T - v(z, \ell).$$

With $T = \theta wz$ and $v = z^2/2\gamma\ell$, this yields

$$z = \gamma\ell w(1-\theta)$$

(5) and (6) as follows :

$$z_{t+1} = \bar{z}$$

$$s_t = \frac{\beta\ell}{1+\beta\ell}w_t - \frac{\bar{z}}{R_{t+1}(1+\beta\ell)}(w_{t+1} - \bar{z}/2\gamma\ell) - \tau \left(\frac{\beta\ell}{1+\beta\ell} + \frac{1+n}{(1+\beta\ell)R_{t+1}} \right) \quad (7)$$

Production side of the economy is characterized by a Cobb-Douglas production function:

$$Y_t = F(K_t, N_t) = AK_t^\alpha N_t^{1-\alpha} \quad (8)$$

where K is the stock of capital, A is a productivity parameter, and N is the labor force. We distinguish N_t the labor force and L_t the size of generation t . The labor force comprises the young population of generation t and the labor force participation from the old generation $t-1$. Incorporating the population growth, $L_t = L_{t-1}(1+n)$, the labor force can be written as $N_t = L_t + L_{t-1}z_t = L_{t-1}(1+n+z_t)$. In comparison, total population at time t is :

$$L_t + \ell L_{t-1} = L_{t-1}(1+\ell+n).$$

Denoting $K_t/N_t \equiv k_t$ and $Y_t/N_t \equiv y_t$, we obtain the income per worker (and not per capita):

$$y_t = f(k_t) = Ak_t^\alpha$$

Factors of production are paid according to their marginal contributions :

$$R_t = f'(k_t) = A\alpha k_t^{\alpha-1} \quad (9)$$

$$w_t = f(k_t) - f'(k_t)k_t = (1-\alpha)Ak_t^\alpha \quad (10)$$

In the case of optimal distortionless retirement,

$$z = z^* = \gamma\ell w$$

In the case of induced early retirement,

$$z = \bar{z} = \gamma\ell w(1-\theta)$$

where θ is chosen such as to generate $\bar{z} < z^*$.

while the equilibrium conditions in the labor and capital markets are as follows:

$$N_t = L_{t-1} (1 + n + z_t) \quad (11)$$

$$K_{t+1} = L_t s_t \quad (12)$$

where the latter expression reflects the fact that the capital is assumed to be depreciated completely after each period. Although, this assumption arises from convenience, it is not unrealistic considering the fact that a period denotes several decades in calendar. Using the optimality condition for savings derived before, the latter expression can be rewritten as follows :

$$\begin{aligned} G_t \equiv & (1 + n + z_{t+1}) k_{t+1} - \frac{\beta \ell}{1 + \beta \ell} A(1 - \alpha) k_t^\alpha + \tau \left(\frac{\beta \ell}{1 + \beta \ell} + \frac{(1 + n) k_{t+1}^{1-\alpha}}{A\alpha (1 + \beta \ell)} \right) \\ & + \frac{z_{t+1} k_{t+1}^{1-\alpha}}{(1 + \beta \ell) A\alpha} \left(A(1 - \alpha) k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma \ell} \right) = 0 \end{aligned} \quad (13)$$

which defines the dynamic behavior of capital stock explicitly. The standard (Diamond) case with no social security and work in second period of life can be deduced by shutting down the corresponding sections, $z = \tau = 0$, which generates the following:

$$G_t \equiv (1 + n + z_{t+1}) k_{t+1} - \frac{\beta \ell}{1 + \beta \ell} A(1 - \alpha) k_t^\alpha \quad (14)$$

Comparing (13) and (14) we have two main differences :

- The third term on the right hand side of (13) denotes the double burden that the PAYG imposes to saving.
- The fourth term reflects the double effect of working in the second period: a distortionary effect if z is not optimal and a saving inducement if $z < z^*$.

In equations (6) and (7) we assumed a pension system that relies on a defined contribution (DC) formula in which the tax $\bar{\tau}$ is given and thus the benefits p has to follow through based on demographic shifts. Two alternative systems can also be considered. The first one provides a defined benefit (DB) \bar{p} over the second period: the contribution rate is then endogenous. The other is a scheme which offers constant annuities \bar{a} (DA) during retirement. The three revenue constraints that these systems imply are as follows:

$$DC : \bar{\tau}(1 + n) = p$$

$$DB : \tau(1 + n) = \bar{p}$$

$$DA : \bar{a}(\ell - z) = \tau(1 + n),$$

Table 1: 6 types of social security regimes

	DC	DB	DC
$z = \bar{z} < z^*$	1	2	3
z^*	4	5	6
$z^* = \gamma \ell w.$			

where an upper bar denotes the defined variable, \bar{a} is the defined annuity and τ has to adjust to variations in z , ℓ and n in this case. Note that for each type of pension system, the individual utility has to adjust accordingly. We have thus two characteristics for a social security system : is it DB, DC, DA and does it comprises an early age or optimal age of retirement ? Altogether, these two characteristics, with three and two alternatives in each, respectively, provide six different ways to describe the equation (13). The six cases are presented on Table 1, and the corresponding equations for G_{it} are provided in appendix.

Using these six alternative specifications, we now turn to the comparative statics of the problem. But before let us state that we assume that the dynamics of capital accumulation (eq(13)) lead to a unique and stable equilibrium, which implies that $0 < \frac{\partial k_{t+1}}{\partial k_t} < 1$. This condition implies that in the steady state $G_k = \frac{\partial G}{\partial k} > 0$.

3 Comparative Statics

In this section, we investigate the comparative statics for the six alternative cases of social security systems identified in the previous section. Our main aim is to elaborate on the behavior of capital accumulation when the economy experiences aging due to lower fertility or higher longevity. We begin by showing the impact of a decrease in fertility in an early retirement system:

$$\Delta \frac{\partial k^1}{\partial n} = -k - \frac{\bar{\tau} k^{1-\alpha}}{A\alpha(1+\beta\ell)} < 0 \quad (15)$$

$$\Delta \frac{\partial k^2}{\partial n} = -k + \frac{\bar{p}\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (16)$$

$$\Delta \frac{\partial k^3}{\partial n} = -k + \frac{\bar{a}(\ell-z)\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (17)$$

where $\Delta = \left(\frac{\partial G}{\partial k}\right)^{-1} > 0$, and superscripts denote the type of social security as defined in Table 1. In a standard case (Diamond), an increase in fertility has a depressive effect on capital accumulation in the absence of a PAYG pension system. This is shown by the first term on the right hand side of each equation above. This depressive effect is reinforced with a DC pension system as shown by the negative second term in (15), but it is weakened or possibly reversed with DB or DA pensions as shown by positive second terms in (16) and (17).

Next, we turn to the impact of an increase in longevity on equilibrium capital per worker in an induced early retirement system:

$$\Delta \frac{\partial k^1}{\partial \ell} = \frac{1}{(1 + \beta\ell)^2} [Ak^\alpha(1 - \alpha)\beta - \beta\bar{\tau} \left(1 - \frac{k^{1-\alpha}(1 + n)}{A\alpha}\right) - \frac{\bar{z}k^{1-\alpha}}{2A\alpha\gamma\ell^2} (\bar{z}(1 + 2\beta\ell) - 2\gamma\beta Ak^\alpha\ell^2(1 - \alpha))] \geq 0 \quad (18)$$

$$\Delta \frac{\partial k^2}{\partial \ell} = \frac{1}{(1 + \beta\ell)^2} [Ak^\alpha(1 - \alpha)\beta - \beta\bar{p} \left(\frac{1}{1 + n} - \frac{k^{1-\alpha}}{A\alpha}\right) - \frac{\bar{z}k^{1-\alpha}}{2A\alpha\gamma\ell^2} (\bar{z}(1 + 2\beta\ell) - 2\gamma\beta Ak^\alpha\ell^2(1 - \alpha))] \geq 0 \quad (19)$$

$$\Delta \frac{\partial k^3}{\partial \ell} = \frac{1}{(1 + \beta\ell)^2} [Ak^\alpha(1 - \alpha)\beta - \bar{a} \left(\beta(\ell - \bar{z}) \left(\frac{1}{1 + n} - \frac{k^{1-\alpha}}{A\alpha}\right) + (1 + \beta\ell) \left(\frac{k^{1-\alpha}}{A\alpha} + \frac{\beta\ell}{1 + n}\right)\right) - \frac{\bar{z}k^{1-\alpha}}{2A\alpha\gamma\ell^2} (\bar{z}(1 + 2\beta\ell) - 2\gamma\beta Ak^\alpha\ell^2(1 - \alpha))] \geq 0 \quad (20)$$

A small increase in longevity has a fostering effect on capital accumulation in the absence of pension and work in the second period. When these features are introduced, however, this effect is diminished and could even be reversed, especially in the case of defined annuities. An increase in longevity magnifies the cost of mandatory retirement and the cost of the pension system in all cases.

We now relax the early retirement assumption, and analyze the impact of aging on capital accumulation when retirement is chosen optimally. The analysis here shows that, compared to the case with early retirement, the ability to adjust the retirement age optimally leads to less distortion in equilibrium, but it also diminishes

the incentives for saving.

$$\Delta \frac{\partial k^4}{\partial n} = -k - \frac{\bar{\tau} k^{1-\alpha}}{A\alpha(1+\beta\ell)} < 0 \quad (21)$$

$$\Delta \frac{\partial k^5}{\partial n} = -k + \frac{\bar{p}\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (22)$$

$$\Delta \frac{\partial k^6}{\partial n} = -k + \frac{\bar{a}\beta\ell^2(1-\gamma D)}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (23)$$

Similar to the case with early retirement, the PAYG pension system reinforces the depressive effect of an increase in fertility on capital accumulation in the DC case and weakens or possibly reverses it in the DB or DA cases. Turning to the effect of longevity when z is endogenous, we have:

$$\begin{aligned} \Delta \frac{\partial k^4}{\partial \ell} &= \frac{1}{(1+\beta\ell)^2} [Ak^\alpha(1-\alpha)\beta - \beta\bar{\tau} \left(1 - \frac{k^{1-\alpha}(1+n)}{A\alpha}\right) \\ &\quad - \frac{Ak^{1+\alpha}(1-\alpha)\gamma}{2\alpha} (2\alpha(1+\beta\ell)^2 + (1-\alpha))] \geq 0 \end{aligned} \quad (24)$$

$$\begin{aligned} \Delta \frac{\partial k^5}{\partial \ell} &= \frac{1}{(1+\beta\ell)^2} [Ak^\alpha(1-\alpha)\beta - \beta\bar{p} \left(\frac{1}{1+n} - \frac{k^{1-\alpha}}{A\alpha}\right) \\ &\quad - \frac{Ak^{1+\alpha}(1-\alpha)\gamma}{2\alpha} (2\alpha(1+\beta\ell)^2 + (1-\alpha))] \geq 0 \end{aligned} \quad (25)$$

$$\begin{aligned} \Delta \frac{\partial k^6}{\partial \ell} &= \frac{1}{(1+\beta\ell)^2} [Ak^\alpha(1-\alpha)\beta - Dk\gamma \left((1+\beta\ell)^2 + \frac{\bar{a}}{2A\alpha k^\alpha} + \frac{(1-\alpha)}{2\alpha} \right) \\ &\quad - \bar{a}(1-D\gamma) \left(\left(\frac{k^{1-\alpha}}{A\alpha} + \frac{\beta\ell}{1+n} \right) + \beta\ell \frac{(1+\beta\ell)}{1+n} \right)] \end{aligned} \quad (26)$$

$$(27)$$

where $D = (A(1-\alpha)k^\alpha - \bar{a}) > 0$. These equations show that both pension and retirement terms weaken the positive effect of an increase in ℓ on equilibrium value of k for the three types of pensions systems.

The analysis so far has shown that demographic change has different implications for capital accumulation under alternative PAYG systems. Analytically, these results are sufficient to show that the impacts are different quantitatively. In order to better grasp these effects, we employ a numerical example. To this effect, we use a set of parameter values to characterize the initial steady state. They are described in the following table.

$A = 10$	$\alpha = 0.33$	$\gamma = 0.05$	$\bar{z} = 0.05$
$\beta = 0.25$	$n = 0.2$	$\ell = 0.6$	$\tau = 1$

Using these values we simulate the equilibrium profiles for the agent's lifetime utility U and the capital per worker k with different values of fertility n and longevity ℓ . We present these profiles in the figures below.

Our numerical illustration indicates that in most cases the incidence of aging has the same sign as without pension or activity in the second period. However, there are exceptions. Let us consider the early retirement case illustrated in Figure 1. An increase in fertility may have a positive effect on utility for low rates of fertility in case of defined benefits or defined annuities. This means that the decline in capital accumulation is more than offset by the fact that the pension burden is alleviated by an increase in fertility. As to longevity increase, we see that in the case of DA it can have a depressive effect on welfare when longevity is high enough. The reason is to be found in the fact that the age of retirement increases as well, which induces a loss in utility. Turning to the case of optimal retirement, we again find that utility increases as n increases under DB or DA regimes. When longevity increases and social security is of the DA type, utility decreases for high levels of longevity. People living longer implies more disutility of work and less consumption in the second period.

4 Dynamics

We now focus on the dynamic analysis of the problem. We are particularly interested by the gains (if any) in capital accumulation and in utility resulting from aging and by the cases where the transition can be welfare worsening for some generations. In other words, we investigate if there are any cases where the short term impact of demographic shift contradicts the long term implications.

The simulations in this section use the same parameter values as in the static simulations in the previous section. However, in this case, we need to specify the magnitude of demographic transition between two steady states as a single value. In order to make comparable the changes in n and ℓ , we assume that a fertility driven aging emerges from a decrease in n from 0.2 to 0.1, and a longevity driven aging arises from an increase in ℓ from 0.8 to 0.873. Both changes imply the same magnitude of aging measured by the increase in the rate of dependency $\frac{\ell}{1+n}$, (about 9 percent increase from the initial steady state dependency ratio), and both changes

Figure 1: Equilibrium Values of Life-time Utility and Capital Per Worker under Alternative PAYG Systems with Early Retirement

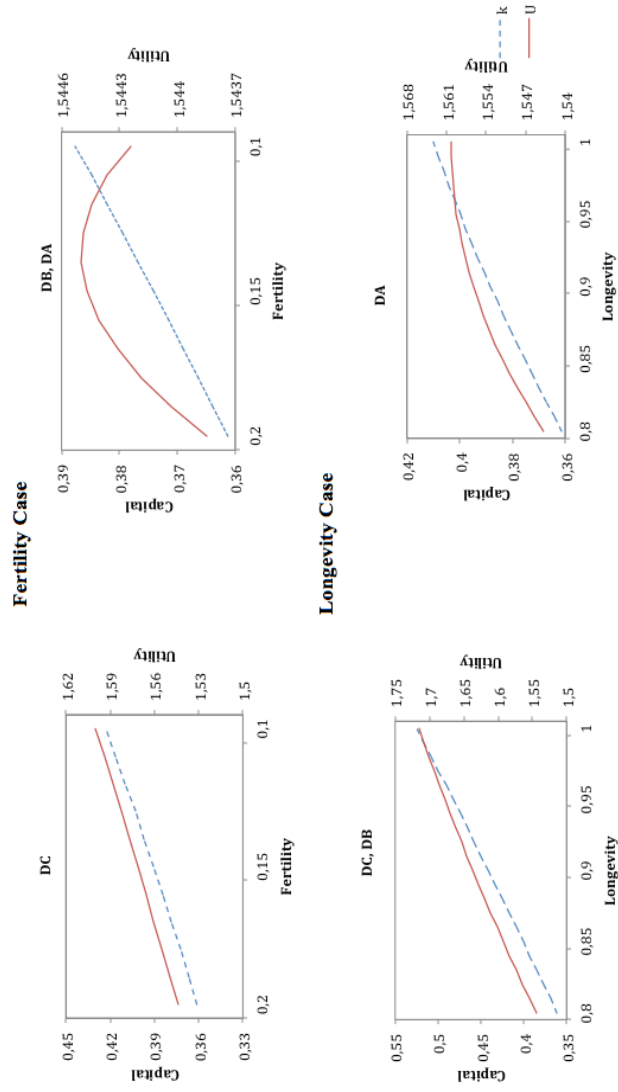


Figure 2: Equilibrium Values of Life-time Utility and Capital Per Worker under Alternative PAYG Systems with Optimal Retirement: Fertility case.

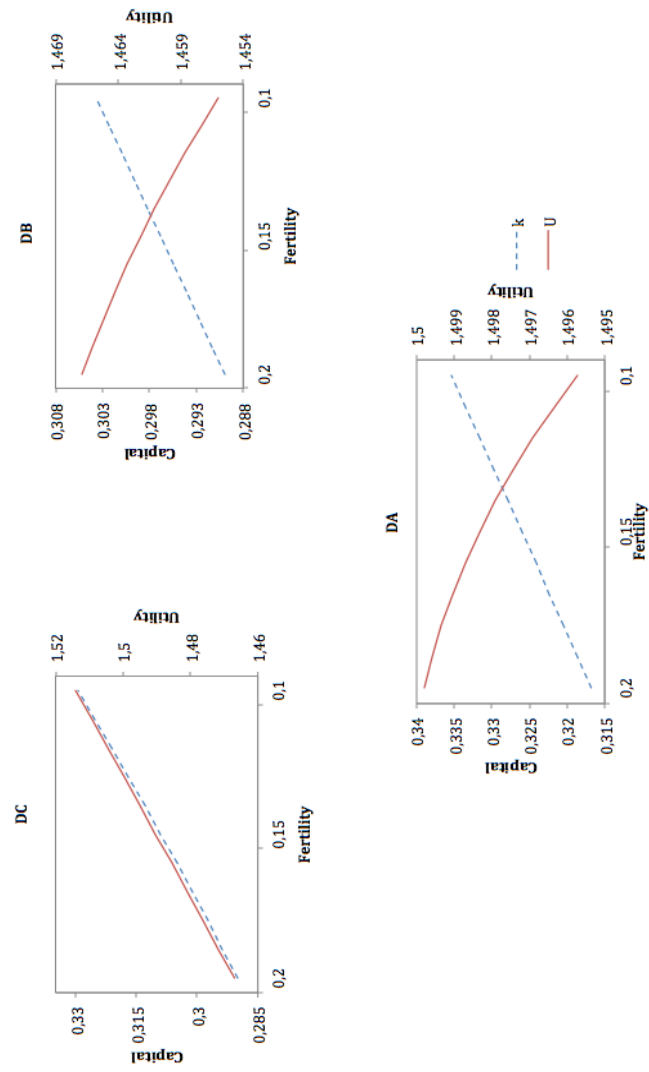
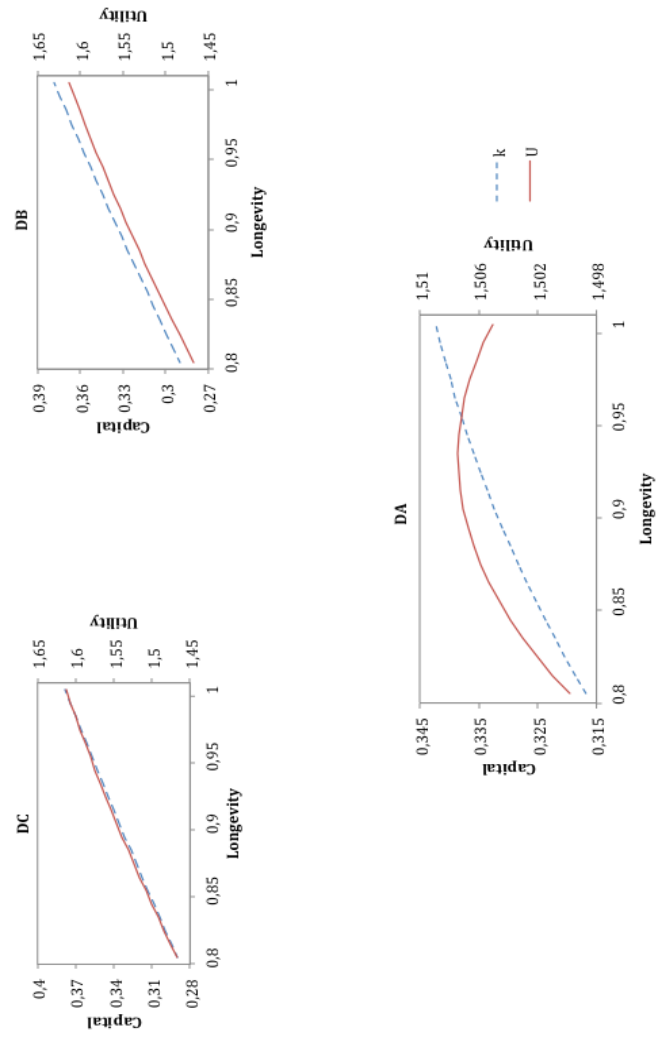


Figure 3: Equilibrium Values of Life-time Utility and Capital Per Worker under Alternative PAYG Systems with Optimal Retirement: Longevity case.



are anticipated by the agents. These demographic shocks leads to new equilibriums over time. Table 2 presents the changes in the steady values of capital per worker and lifetime utility in percentage terms. Both types of aging have the most positive impact on capital accumulation and lifetime utility under a defined contribution system. The least positive impact, or the most negative in the case of fertility shock, alternates between defined benefits and defined annuities depending on the type of aging.

The transition dynamics for these shocks are illustrated by the Figures 4 and 5, where a permanent demographic shock is introduced in period 6.

Table 2: Comparative Statics for Capital per Worker and Lifetime Utility under Different Social Security Systems and Shock Scenarios
(Percentage Change in the Steady State Values after the Shock)

	Longevity Shock		Fertility Shock	
	Capital per Worker	Lifetime Utility	Capital per Worker	Lifetime Utility
Early Retirement				
Defined Contribution	16.22	4.64	17.37	3.79
Defined Benefit	16.22	4.64	7.31	0.03
Defined Annuity	5.59	0.63	7.31	0.03
Optimal Retirement				
Defined Contribution	11.75	4.17	13.87	3.37
Defined Benefit	11.75	4.17	4.73	-0.76
Defined Annuity	3.72	0.40	5.88	-0.27

The most striking result is the transitory loss in utility following a fertility shock, particularly with DC. This is due to substantial loss in old age consumption when the shock hits. Intuitively, a lower fertility in period 6 reduces the retirement benefits for the old generation within the same period under the defined contributions system. Anticipating this reduction, the generation that is born in period 5 increases the savings as implied by the Euler equation. In equilibrium, this increase in savings joins the reduction in the labor force to increase the real wages in period 6, however this increase is not sufficient to fully compensate the decrease in retirement benefits. As a result, the generation that is born in period 5, who constitute the old generation when the demographic shock hits in period 6, experiences a loss in lifetime utility. This transitory cost does not appear in the case of an aging driven by increasing longevity.

From Table 2 and Figures 4 and 5, it appears that the defined contribution formula dominates the other formulas both in capital increase and utility gain as

Figure 4: Dynamic path: Early Retirement

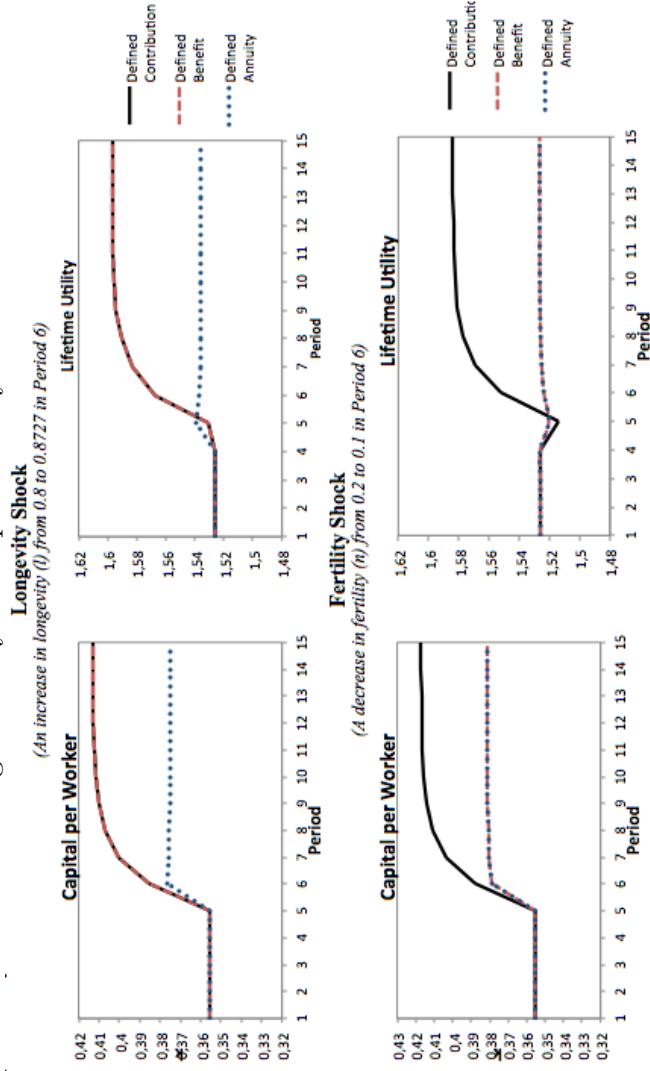
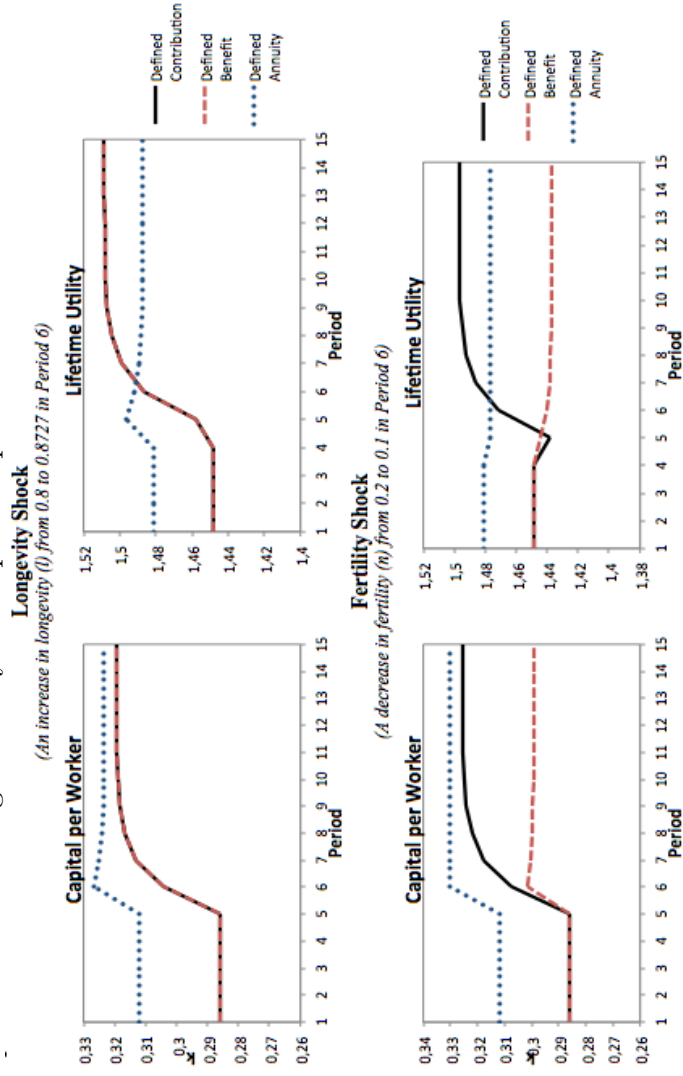


Figure 5: Dynamic path: Optimal Retirement



a response to aging in the steady state. Induced early retirement seems also to dominate flexible retirement meaning that the labor distortion is more than offset by the gain in capital accumulation.

5 Conclusions

In this paper we have tried to evaluate the implications of different sources of aging on both the level of capital and of welfare of economies having social security systems that can differ in three ways: defined benefits or contributions, funded or not, early retirement or flexible retirement. We show that the effects of longevity increase and fertility decrease, two phenomena that contribute to aging, vary in a contrasted way depending on these features of the pension system. This is important as we know that countries do not age the same way and that they tend to shift progressively from a regime of defined benefits towards one of defined contributions. On our further research agenda, we intend to look at the joint effect of aging and changes in the social security regimes: from DA/DB to DC and from early retirement to flexible retirement. Another limitation of the the current analysis is that individuals are all identical. With heterogeneity in wages, one would find more merits in the defined annuity formula.

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Appendix

$$G_{1t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{\tau}}{1+\beta\ell} \left(\beta\ell + \frac{(1+n)k_{t+1}^{1-\alpha}}{A\alpha} \right) \quad (28)$$

$$+ \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (29)$$

$$G_{2t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{p}}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (30)$$

$$G_{3t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{a}(\ell - \bar{z})}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (31)$$

$$G_{4t} \equiv (1+n+A(1-\alpha)k^\alpha\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha \\ + \frac{\bar{\tau}}{1+\beta\ell} \left(\beta\ell + \frac{(1+n)k_{t+1}^{1-\alpha}}{A\alpha} \right) + \frac{A^2(1-\alpha)^2\gamma\ell k_{t+1}^{1+\alpha}}{2(1+\beta\ell)A\alpha} = 0 \quad (32)$$

$$G_{5t} \equiv (1+n+A(1-\alpha)k^\alpha\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha \\ + \frac{\bar{p}}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) + \frac{A^2(1-\alpha)^2\gamma\ell k_{t+1}^{1+\alpha}}{2(1+\beta\ell)A\alpha} = 0 \quad (33)$$

$$G_{6t} \equiv (1+n+D_t\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{a}(\ell - D_t\gamma\ell)}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{D_t\gamma\ell k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{D_t}{2} \right) = 0$$

Where $D_t = (A(1-\alpha)k_t^\alpha - \bar{a})$

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